647510 **MSC1MatC104x** **Seat No :________**

M.Sc.(Maths.) Semester - 1 (*CBCS*) Examination

Nov/Dec-2023

Theory of Ordinary Differential Equations(Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. Figures to the right indicate marks.
2. All questions are compulsory.

Que.1(A) Answer any Two out of Three. (10)

1) Solve the initial value problem $y' = \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix} y$ with $y(t_0) = \begin{bmatrix} y_{01} \\ y_{02} \end{bmatrix}, \forall\, t_0, t \in \mathbb{R}$.
2) Define system of first order linear ordinary differential equations. Write the differential equation $y_1' = y_2 + \cos t\,, y_2' = y_1$ in the matrix form.
3) Show that $u(t) = \begin{bmatrix} \cos t \\ -\sin t \end{bmatrix}$ is a solution of the initial value problem $y' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} y,\ \ y(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ on $\mathbb{R}$.

Que.1(B) Answer any One out of Two. (04)

1) Define order and degree of differential equation with example.
2) Reduce $y'' + 2y' + 7ty = e^{-t}, y(1) = 7, y'(1) = 2$ to initial value problem of first order linear differential equation.

Que.2(A) Answer any Two out of Three. (10)

1) Let A be an $n \times n$ real matrix and $\lambda_1, \lambda_2, \dots, \lambda_n$ be eigen values of A and $v_1, v_2, \dots, v_n \in K^n$ be an eigen vectors corresponding to eigen values $\lambda_1, \lambda_2, \dots, \lambda_n$ respectively then
$$\emptyset(t) = \begin{bmatrix} e^{\lambda_1} v_1 & e^{\lambda_2} v_2 & \dots & e^{\lambda_n} v_n \end{bmatrix}, \forall\, t \in \mathbb{R}$$
is a fundamental matrix of $y' = Ay$ on $\mathbb{R}$.
2) Find the fundamental matrix of $y' = Ay$ on $\mathbb{R}$. Where $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.
3) Define exponential of an $n \times n$ matrix. Let A and B be two $n \times n$ real matrix then $e^{A+B} = e^A\, e^B$ if $AB = BA$.

Que.2(B) Answer any One out of Two. (04)

1) Find the general solution of $y''' + 3y'' + 3y' + y = 0$ on $\mathbb{R}$.
2) Prove that e^{3t} and te^{3t} are LI solutions of $y'' - 6y' + 9y = 0$ on $\mathbb{R}$.

Que.3(A) Answer any Two out of Three. (10)

1) Obtain solution in terms of power series near origin of the differential equation $y'' - xy' - y = 0$.
2) Classify the singularities at the point infinity of the differential equation $x^4 y'' + x^3(x+2)y' + y = 0$.
3) Show that $\frac{d}{dx}[x^\alpha J_\alpha(x)] = x^\alpha J_{\alpha-1}(x)$ and $\frac{d}{dx}[x^{-\alpha} J_\alpha(x)] = -x^{-\alpha} J_{\alpha+1}(x)$.

Que.3(B) Answer any One out of Two. (04)

1) Find out set of ordinary points and set of singular points of the differential equation
$$x(x-1)^2(x+2)y'' + x^2(x-1)y' + (x+2)y = 0.$$
2) Define radius of convergence of power series and find the radius of convergence of power series $\sum_{n=0}^{\infty} \frac{x^n}{n}$.

Que.4(A) Answer any Two out of Three. (10)

1) State and prove Able's formula.
2) State and prove variation of constant formula for scalar linear second order non-homogenous differential equation.
3) Let A be a constant 2×2 complex matrix and λ and μ two different eigen values of A then prove that there exists a constant 2×2 non-singular matrix T such that $T^{-1}AT$ has the form $\begin{bmatrix} \lambda & 0 \\ 0 & \mu \end{bmatrix}$.

Que.4(B) Answer any One out of Two. (04)

1) If y_1, y_2 are solution of $(1-x^2)y'' - 2xy' + p(p-1)y = 0$ with initial condition $y_1(0) = 0, y_1'(0) = -1, y_2(0) = 1, y_2'(0) = 0$ then find the Wronskian of $W(y_1, y_2)\left(\frac{1}{2}\right)$.
2) Let A be any $n \times n$ real constant matrix then $\emptyset(t) = \exp(tA)$, $t \in \mathbb{R}$ is a fundamental matrix of $y' = Ay$ with $\emptyset(0) = I$ on $\mathbb{R}$.

Que.5(A) Answer any Two out of Three. (10)

1) State and prove convolution theorem for Laplace transform.
2) Solve $y'' + y = 2e^t$ subject to $y(0) = 2, y'(0) = 2$ using Laplace transformation.
3) Compute the Laplace transform of $\frac{1-\cos 2t}{t}$ and $\frac{\cos 2t - \cos 3t}{t}$.

Que.5(B) Answer any One out of Two. (04)

1) Find $L(\sin ct)$, $c \in \mathbb{C}$.
2) Find $L^{-1}\left(\frac{3z+7}{z^2-2z-3}\right)$.

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